Cerenkov Radiation in Anisotropic Ferrites*

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Summary—A numerical analysis and an experimental study of Cerenkov radiation from an anisotropic ferrite are reported in this paper. Extensive curves of the interaction resistance R_λ, measured per unit wavelength of interaction distance, are presented as functions of the several ferrite and geometric parameters. Values in excess of 10^3 ohms per wavelength of interaction distance are noted. The conditions for Cerenkov radiation in the ferrite are derived from consideration of plane-wave propagation through the ferrite. X-band output powers of approximately one watt were observed using a 0.88 Mev bunched electron-beam with a peak current of 18 ma.

I. Introduction

THE CERENKOV effect as a means of generating microwaves was proposed by Ginsburg[1] in 1947. Many contributions to the theoretical literature were precipitated by this suggestion; however, experimental papers on the generation of coherent microwave energy via the Cerenkov effect have appeared only recently.[2–5] There thus exists the peculiar situation of an effect which appears to be well understood but as yet unevaluated in the practical sense.

Experiments with this interaction in an isotropic dielectric medium[3] have yielded radiated powers of the order of 0.5 watt at a wavelength of 8.31 mm. The possibility of enhancement of the Cerenkov interaction exists in using media which exhibit characteristic frequencies or resonances.

In this paper the Cerenkov interaction in a ferrite is considered. This material provides a highly dispersive, anisotropic, low loss medium for study in the microwave range. In the low millimeter range the biasing magnetic field required becomes rather large, which may be a disadvantage. In this case it would be desirable to seek a material with "built-in" fields, for example, an antiferrimagnetic substance.

The effects of the ferrite properties on the Cerenkov radiation extracted from a bunched relativistic electron beam in an anisotropic ferrite are presented numerically and experimentally. The numerical results were obtained with the aid of a digital computer. The detailed mathematical analysis has been treated elsewhere.[6]

II. Fundamental Relations

The total current density, J, of the bunched electron-beam may be represented as the sum of the harmonic current densities as follows

$$J = \hat{\imath}_z v_0 \sum_{n=-\infty}^{\infty} \rho_n e^{j\omega_n (t - z/v_0)} \tag{1}$$

where $\hat{\imath}_z$ is the unit vector in the z direction; v_0 is the dc beam velocity; ρ_n is the harmonic charge density; ω_n is the nth-harmonic radian frequency. Energy may be extracted at any given harmonic frequency, so the Cerenkov frequency spectrum will contain maxima corresponding to these beam harmonics. Since we will generally refer to only one harmonic, the subscripts will be dropped. No velocity spread in the beam is considered.

Ferrite microwave properties are described by the Polder[7] permeability tensor

$$\overline{\overline{\mu}} = \mu_0 \begin{bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2a}$$

where

$$\mu = \frac{\frac{f_c}{f}\left(\frac{f_c}{f} + \frac{f_m}{f}\right)(1+\alpha^2) - 1 + j\alpha\left(2\frac{f_c}{f} + \frac{f_m}{f}\right)}{\left(\frac{f_c}{f}\right)^2 (1+\alpha^2) - 1 + j\alpha 2 \frac{f_c}{f}} \tag{2b}$$

$$\kappa = \frac{\frac{f_m}{f}}{\left(\frac{f_c}{f}\right)(1+\alpha^2) - 1 + j\alpha 2 \frac{f_c}{f}} \tag{2c}$$

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[1] V. L. Ginzburg, "Utilization of the Cherenkov effect for producing emission of radio waves," translated from *Dokl. Akad. Nauk* (SSSR), vol. 56, pp. 253–254; 1947.

[2] M. Danos, S. Geshwind, H. Lashinsky and A. Van Trier, "Cerenkov effect at microwave frequencies," *Phys. Rev.*, vol. 92, p. 828; November, 1953.

[3] P. Coleman and C. Enderby, "Megavolt electronics Cerenkov coupler for the production of millimeter and submillimeter waves," *J. Appl. Phys.*, vol. 31, pp. 1695–1696; September, 1960.

[4] IU. V. Anisimova, G. A. Bernashevskii, A. N. Vystavkin and L. G. Lomize, "Millimeter-band investigation of waveguide radiators excited by relativistic electron flows," *Radioteki. i Elecktron.*, vol. 5, pp. 126–142; March, 1960.

[5] L. G. Lomize, "Comparative features of Cerenkov radiation, transition radiation and bremstsrahlung in the microwave region," *Sov. Phys.—Tech. Phys.*, vol. 6, pp. 215–221; September, 1961.

[6] F. Rosenbaum, "Cerenkov radiation from anisotropic ferrites," Ultramicrowave Group, University of Illinois, Urbana, Tech. Note No. 8 of contract No. AF33(616)-10224. This information is also available in the three annual reports (March, 1961, December, 1961, September 1962) of contract No. AF33(616)-7043, Ultramicrowave Group, University of Illinois, Urbana.

[7] D. Polder, "On the theory of ferromagnetic resonances," *Phil. Mag.*, vol. 40, p. 99; 1949.

and

f = given harmonic frequency
$f_c = \gamma H_i$ = precession frequency
$f_m = \gamma 4\pi M_s$ = magnetization frequency
γ = gyromagnetic ratio $= 2.8 \times 10^6$ cps/oe
H_i = internal dc magnetic field
$4\pi M_s$ = saturation magnetization
α = a measure of the damping.[8]

If the medium is lossless, $\alpha = 0$. We neglect dielectric losses and assume a real dielectric constant ϵ_f for the ferrite.

In this report the time-varying field quantities and the electrical and magnetic properties of the media will be written in the MKS unit system. The CGS unit system will be used for the static magnetic field quantities in accordance with common usage. (For example, see Lax and Button.[9])

Our program is to obtain the field amplitudes either by a Green's function technique or from the solution of a boundary value problem. These amplitudes are used to compute the strength of the Cerenkov interaction from a $(\boldsymbol{J} \cdot \boldsymbol{E}^*)$ or an $(\boldsymbol{E} \times \boldsymbol{H}^*)$ integral. The average power radiated P_{av} thus obtained may be written as

$$P_{av} = 1/2 I_n^2 R_\lambda (L/\lambda) \tag{3}$$

where I_n is the nth harmonic current peak amplitude, R_λ is an interaction resistance per wavelength, and (L/λ) is a number of wavelengths involved in the interaction. The results of this numerical analysis are presented as curves of R_λ as a function of the several parameters.

III. Radial Propagation Constants and Condition for Cerenkov Radiation

Maxwell's equations for the fields in an anisotropic ferrite of infinite extent can be written as

$$\nabla \times \boldsymbol{E} = -j\omega \overline{\overline{\mu}} \cdot \boldsymbol{H} \quad (4a) \qquad \nabla \cdot \boldsymbol{B} = 0 \quad (4c)$$

$$\nabla \times \boldsymbol{H} = j\omega\epsilon_0\epsilon_f \boldsymbol{E} \quad (4b) \qquad \nabla \cdot \boldsymbol{D} = 0 \quad (4d)$$

where $\overline{\overline{\mu}}$ is the Polder permeability tensor.

Kales[10] has shown that the transverse field components in the ferrite may be expressed as functions of E_z and H_z with these satisfying

$$\nabla_t^2 E_z + aE_z + bH_z = 0 \tag{5a}$$

$$\nabla_t^2 H_z + cH_z + dE_z = 0 \tag{5b}$$

where ∇_t^2 is the transverse Laplace operator and

$$a = K^2 - k'^2(\kappa/\mu) \quad (5c) \qquad c = K^2/\mu \quad (5f)$$

$$b = -\omega\gamma_z\mu_0(\kappa/\mu) \quad (5d) \qquad d = \omega\gamma_z\epsilon_0\epsilon_f(\kappa/\mu) \quad (5g)$$

$$K^2 = \left(\frac{\omega}{c_0}\right)^2 \epsilon_f\mu + \gamma_z^2 \quad (5e) \qquad k'^2 = \left(\frac{\omega}{c_0}\right)^2 \epsilon_f\kappa \quad (5h)$$

γ_z = propagation constant in z direction

c_0 = velocity of light.

Solutions of (5) have the form[10]

$$E_z = (\sigma_1^2 u_1 + \sigma_2^2 u_2)e^{(j\omega t - \gamma_z z)} \tag{6a}$$

$$H_z = (q_1 u_1 + q_2 u_2)e^{(j\omega t - \gamma_z z)} \tag{6b}$$

where

$$\sigma_{1,2}^2 = \tfrac{1}{2}\{(a + c) \pm \sqrt{(a - c)^2 + 4bd}\} \tag{6c}$$

and

$$q_{1,2} = \frac{\sigma_{1,2}^2(\sigma_{1,2}^2 - a)}{b}. \tag{6d}$$

Here u_1 and u_2 represent the appropriately chosen radial functions necessary to satisfy the radial boundary conditions. In cylindrical coordinates

$$u_{1,2} = A_{1,2}R_m(\sigma_{1,2}r)e^{jm\theta} \qquad m = 0, \pm 1, \pm 2, \cdots \tag{6e}$$

where $R_m(x)$ is the appropriate Bessel function, $A_{1,2}$ are constants and (σ_1, σ_2) are the radial propagation constants. Due to the coupled nature of (5) all field components exist in the ferrite, *i.e.*, a hybrid mode.

Notice that two constants (σ_1, σ_2) appear. These describe the two modes of radial propagation in the ferrite. If either σ_1 or σ_2 is imaginary, that wave will not propagate. From this fact the conditions for Cerenkov radiation may be determined.

Consider

$$\sigma_1^2\sigma_2^2 = ac - bd \tag{7a}$$

$$= \frac{1}{\mu}\left[\left(\frac{\omega}{c_0}\right)^2 \epsilon_f(\mu + \kappa) + \gamma_z^2\right] \cdot \left[\left(\frac{\omega}{c_0}\right)^2 \epsilon_f(\mu - \kappa) + \gamma_z^2\right]. \tag{7b}$$

If $\sigma_1^2\sigma_2^2$ is set equal to zero one obtains the equations for ordinary and extraordinary plane-wave propagation through a lossless ferrite in the direction of the dc magnetic field,[9] *i.e.*,

$$\gamma_z^2 = -\left(\frac{\omega}{c_0}\right)^2 \epsilon_f(\mu \pm \kappa) = -(\bar{\beta}_z)^2. \tag{8}$$

At this value of γ_z^2 one of the radial propagation constants is zero. Eq. (8) represents the $(\omega - \bar{\beta}_z)$ relation for planewave propagation in the ferrite. One such $(\omega - \bar{\beta}_z)$ diagram is shown in Fig. 1.

[8] S. Sensiper, "Resonance loss properties of ferrites in 9 kMc region," Proc. IRE, vol. 44, pp. 1323–1342; October, 1956.
[9] B. Lax and K. Button, "Microwave Ferrites and Ferrimagnetics," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 319–320; 1962.
[10] M. Kales, "Modes in wave guides containing ferrites," *J. Appl. Phys.* vol. 24, pp. 604–608; May 1953.

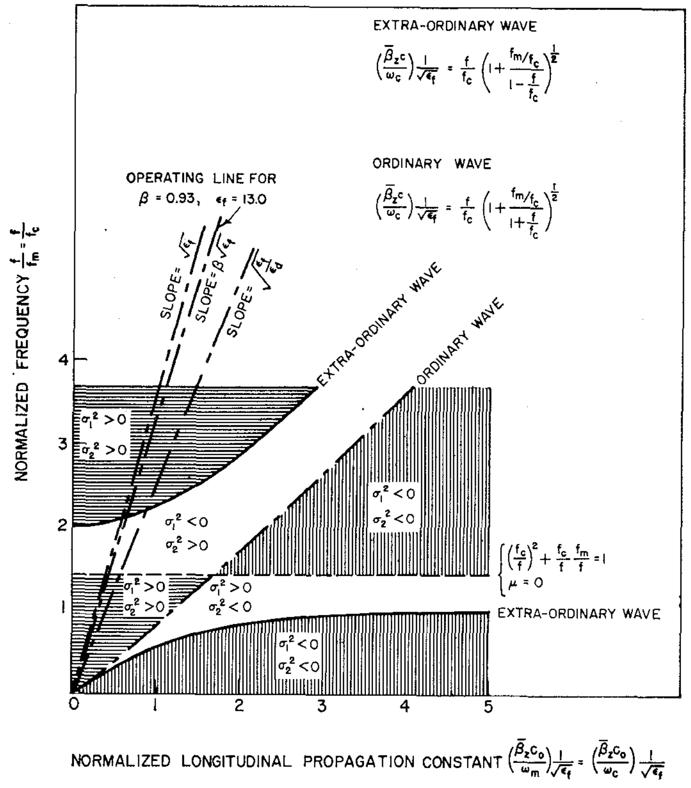

Fig. 1—ω-$\bar{\beta}_z$ diagram for a plane-wave propagating parallel to the applied dc magnetic field in a ferrite with $f_c/f_m = 1$. Cerenkov radiation occurs in region where either σ_1^2, σ_2^2, or both are greater than zero.

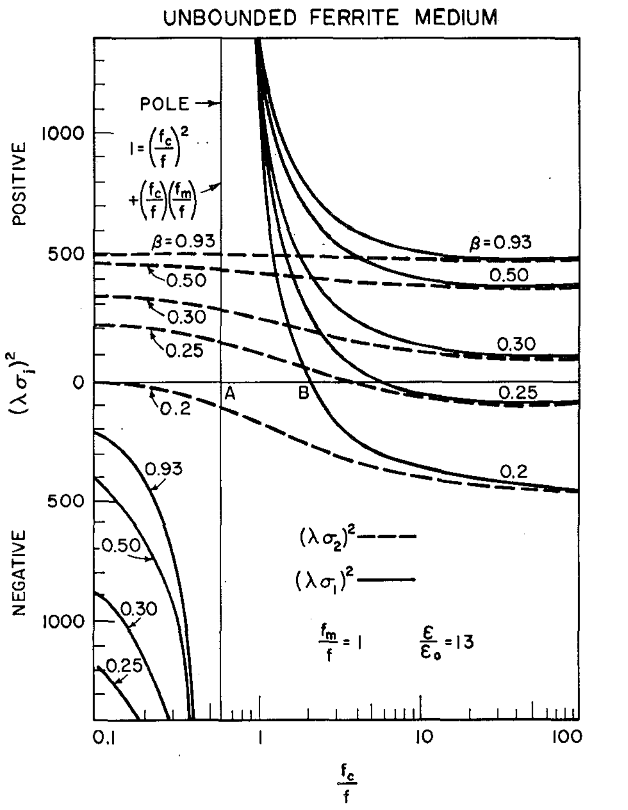

Fig. 2—Normalized radial propagation constants as a function of f_c/f.

The curve $\sigma_1^2\sigma_2^2 = 0$ is the locus of the zeros of σ_1^2 or σ_2^2, *i.e.*, it bounds the regions of various sign possibilities for the squares of the radial propagation constants. Examination of (9) shows that the point $(f_c/f)^2 + (f_c/f)(f_m/f) = 1$ is a singularity for σ_1^2 and that a sign change occurs across this point. If this line is included, the $(\omega - \bar{\beta}_z)$ plane may be used to determine the regions where Cerenkov radiation exists for the ferrite. These regions are also shown in Fig. 1. Since there is no radial propagation for a mode if its σ is imaginary, this condition constitutes a cutoff for Cerenkov radiation in the ferrite. There will be Cerenkov radiation only if some radial propagation is possible.

To extract microwave energy from the bunched beam one must match a beam harmonic with the ferrite fields. The beam fields go as $e^{j\omega t - \gamma_z z}$ and $\gamma_z = j\omega/v_0$. This condition defines an operating line in the $(\omega - \bar{\beta}_z)$ plane with slope $\beta\sqrt{\epsilon_f}$ ($\beta = v_0/c_0 =$ relative beam velocity). In an unbounded ferrite the slope of the operating line may range from zero to $\sqrt{\epsilon_f}$. A given set of parameters specifies the point on this line where the beam harmonic is matched to the ferrite fields.

If the ferrite has finite radius and is surrounded by an unbounded dielectric medium of constant ϵ_d, the condition for Cerenkov radiation to propagate in this dielectric is $\sqrt{\epsilon_f} \geq \beta\sqrt{\epsilon_f} \geq \sqrt{\epsilon_f/\epsilon_d}$, and the Cerenkov angle in the dielectric is $\cos\theta_c = 1/\beta\sqrt{\epsilon_d}$.

In terms of the lossless ferrite parameters the square of the radial propagation constants may be written as follows:

$$(\sigma_{1,2}\lambda)^2 = \frac{2\pi^2}{\left(\frac{f_c}{f}\right)^2 + \left(\frac{f_c}{f}\right)\left(\frac{f_m}{f}\right) - 1}\left\{2\left[\left(\frac{f_c}{f}\right)^2 - 1\right]\left(\epsilon_f - \frac{1}{\beta^2}\right) + \epsilon_f\left(\frac{f_m}{f}\right)\left[\frac{f_c}{f}\left(3 - \frac{1}{\beta^2\epsilon_f}\right) + \frac{f_m}{f}\right] \pm \sqrt{\left(\frac{f_m}{f}\right)^2\left[\epsilon_f\left(\frac{f_c}{f} + \frac{f_m}{f}\right) - \left(\frac{f_c}{f}\right)\frac{1}{\beta^2}\right]^2 + \left(\frac{f_m}{f}\right)^2\frac{4\epsilon_f}{\beta^2}}\right\}. \quad (9)$$

The plus sign corresponds to $(\sigma_1\lambda)^2$, the minus to $(\sigma_2\lambda)^2$.

This is plotted as a function of f_c/f in Fig. 2 for a given set of ferrite parameters. For $\beta = 0.2$ Cerenkov radiation will propagate only for values of f_c/f between points A and B, where σ_1^2 is positive. This explains the sort of behavior predicted by Lashinsky[11] for a beam of $\beta = 0.2$.

[11] H. Lashinsky, "Cerenkov radiation from extended electron beams near a medium of complex index of refraction," *J. Appl. Phys.*, vol. 27, pp. 631–635; June, 1956.

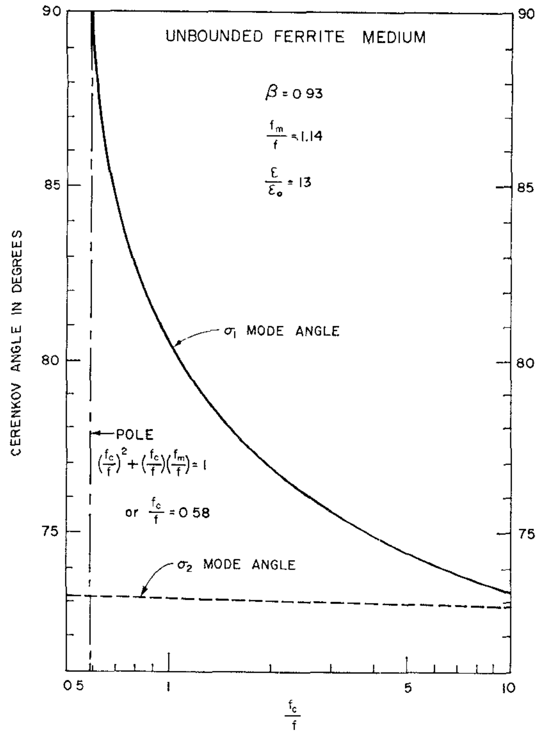

Fig. 3—Cerenkov angles in ferrite as functions of f_c/f.

The Cerenkov angles for the two ferrite propagation modes may be derived as follows. The field components under the far field approximation have phase factors of the form $\exp j[\omega t-(\sigma_i r+\bar{\beta}_z z)]$. Hence propagation is in both the radial and the longitudinal directions. The angle θ at which the plane-wave propagates is then

$$\cos\theta = \frac{\bar{\beta}_z}{(\bar{\beta}_z{}^2+\sigma_i{}^2)^{1/2}} \quad \text{and} \quad \bar{\beta}_z = \frac{\omega}{v_0} = \frac{2\pi}{\beta\lambda}$$

So

$$\cos\theta = \frac{1}{\left[1+\left(\dfrac{\sigma_i\lambda\beta}{2\pi}\right)^2\right]^{1/2}} \leq 1. \tag{10}$$

Using numerical values for $(\sigma_{1,2}\lambda)^2$ determined from (9) it is easy to obtain the behavior of the Cerenkov angles in the ferrite with f_c/f as shown in Fig. 3. As f_c/f approaches the pole point $(f_c/f)^2+(f_c/f)(f_m/f)=1$, $(f_c/f=0.58)$, the angle corresponding to the σ_1 mode approaches 90° for all f_c/f ratios. As f_c/f increases beyond the value 0.58, the σ_1 mode angle decreases until at large f_c/f values it approaches the σ_2 mode angle. At large f_c/f values the ferrite behaves essentially as a medium with $\epsilon_f=13$ and ${}^2\bar{\mu}=\mu_0$ so that for $\beta=0.93$ the Cerenkov angle becomes equal to 72.6°.

IV. Cerenkov Radiation from a Lossless Ferrite

Cerenkov radiation from a ferrite is examined in three geometries in order to include the effect of the ferrite properties as well as of geometrical resonances. These geometries are:

Case A—The unbounded ferrite and filamentary bunched beam,

Case B—The unbounded ferrite and non-zero radius bunched beam,

Case C—The bounded ferrite and nonzero radius bunched beam.

In Cases B and C we assume that the beam fills a hole drilled in the medium. In all three cases the ferrite is assumed lossless and is immersed in a saturating dc magnetic field directed parallel to the beam. Each case will be discussed in turn.

A. One-Region Problem

If we allow a filamentary bunched beam of electrons described by current density $J_z=(I_n/2\pi r)\delta(r)$ to stream through an unbounded ferrite, Kales' coupled equations (5) become[6]

$$\nabla_t{}^2E_z + aE_z + bH_z = jJ_z\left(\frac{a}{\omega\epsilon}\right) \tag{11a}$$

$$\nabla_t{}^2H_z + cH_z + dE_z = jJ_z\left(\frac{d}{\omega\epsilon}\right), \qquad \epsilon = \epsilon_0\epsilon_f. \tag{11b}$$

Eqs. (11) determine the behavior of the longitudinal fields, E_z and H_z, which are produced by the current density J_z. The quantity I_n is the harmonic current amplitude. These equations uncouple if we choose as solutions the fields of (6). A Green's function or particular-integral solution to the uncoupled equations may be obtained for the condition of outward going waves,[12] so that

$$u_1 = \frac{I_n}{4\omega\epsilon}\left[\frac{\sigma_2{}^2d - q_2a}{\sigma_1{}^2q_2 - \sigma_2{}^2q_1}\right]H_0{}^{(2)}(\sigma_1 r)$$

$$= \frac{I_n}{4\omega\epsilon}A_1H_0{}^{(2)}(\sigma_1 r) \tag{12a}$$

$$u_2 = \frac{I_n}{4\omega\epsilon}\left[\frac{q_1a - \sigma_1{}^2d}{\sigma_1{}^2q_2 - \sigma_2{}^2q_1}\right]H_0{}^{(2)}(\sigma_2 r)$$

$$= \frac{I_n}{4\omega\epsilon}A_2H_0{}^{(2)}(\sigma_2 r) \tag{12b}$$

where $H_0{}^{(2)}(x)$ is the Hankel function of the second kind, order zero.

The power propagated at large distances from the beam may be found by evaluating

$$P_{av} = \text{Re}\int (\boldsymbol{E}\times\boldsymbol{H}^*)\cdot d\boldsymbol{S} \tag{13}$$

using the field components derived from the solutions in (12) and the large argument expansions for the Hankel functions. The transverse field components may be

[12] P. Morse and H. Feshbach, "Methods of Theoretical Physics," McGraw-Hill Book Co., Inc., New York, N. Y., p. 826; 1953.

derived as indicated by Kales.[10] The interaction resistance defined in (3) and computed in the far field $(\sigma_{1,2}r \gg 1)$, is

$$R_\lambda = \frac{15}{\epsilon_f}\left\{A_1A_1^*\left[\Sigma_1^2 + \frac{(120\pi)^2}{\epsilon_f}\frac{Q_1Q_1^*}{\Sigma_1^2}\right] + A_2A_2^*\left[\Sigma_2^2 + \frac{(120\pi)^2}{\epsilon_f}\frac{Q_2Q_2^*}{\Sigma_2^2}\right]\right\} \quad (14)$$

where $\Sigma_{1,2}^2 = (\sigma_{1,2}\lambda)^2$ = normalized radial propagation constants, and $Q_{1,2} = q_{1,2}\lambda^2$.

B. Two-Region Problem

A more realistic geometry involves a bunched electron-beam of nonzero radius; so we must provide a hole in the ferrite to allow the beam passage. While making things easier for the beam, the hole complicates our arithmetic, for we are now faced with a boundary-value problem. Since a hybrid mode exists in the ferrite we must solve a four-by-four complex matrix equation for the field amplitudes. This equation results from matching tangential electric and magnetic fields across the boundary.

The E_z field in the beam is

$$E_z = j\left[\frac{v_0\rho}{\omega\epsilon_0} + (\lambda_1 + j\psi_1)\frac{\beta\rho\lambda}{2\pi\epsilon_0}I_0(K_0r)\right] \quad (15)$$

where $K_0^2 = (\omega/v_0)^2(1-\beta^2)$, λ is the harmonic wavelength, $I_0(K_0r)$ is the modified Bessel function of the first kind, and $(\lambda_1 + j\psi_1)$ is the complex field constant to be determined from the solution to the amplitude matrix equation. The interaction resistance computed from

$$P = -\tfrac{1}{2}\,\mathrm{Re}\int J\cdot E^* dV$$

is

$$R_\lambda = \frac{120 I_1(K_0R_b)}{\pi\left(\frac{R_b}{\lambda}\right)^3 K_0}\psi_1 \quad (16)$$

with R_b = beam hole radius.

C. Three-Region Problem

Most easily duplicated experimentally is a geometry which consists of a ferrite tube surrounded by an unbounded isotropic dielectric medium of constant ϵ_d. Boundary effects such as feedback due to reflections and coupling between the two modes of propagation now enter the problem. An eight-by-eight complex matrix equation must be solved for the field amplitudes.

The power radiated is examined in the unbounded dielectric. Such an isotropic dielectric medium will not support a hybrid mode; so the ferrite hybrid mode decouples at the boundary and a TE and a TM mode

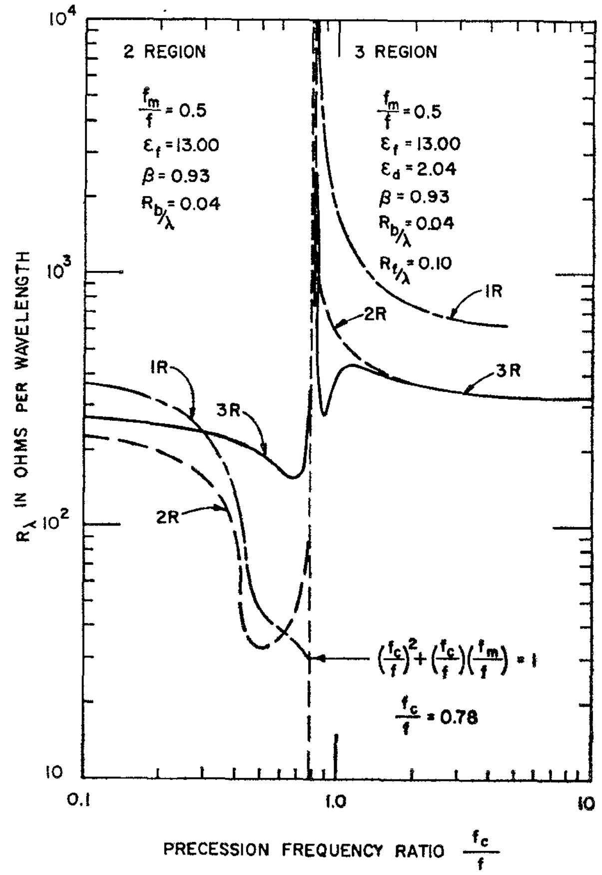

Fig. 4—Comparison of one-, two-, and three-region interaction resistances for given set values of parameters. The existence of the beam tunnel reduces the level of the two-region interaction. Reflections from the ferrite-dielectric interface modify this effect in the three-region interaction.

propagate in the dielectric medium. These modes are TM and TE with respect to the z direction. Their respective interaction resistances are

$$R_{\lambda\mathrm{TM}} = \frac{60\epsilon_d}{\pi^4\left(\frac{R_b}{\lambda}\right)^4\left(\epsilon_d - \frac{1}{\beta^2}\right)}(\lambda_7^2 + \psi_7^2)\frac{\text{ohms}}{\text{wavelength}} \quad (17a)$$

$$R_{\lambda\mathrm{TE}} = \frac{60(120)^2}{\pi^2\left(\frac{R_b}{\lambda}\right)^4\left(\epsilon_d - \frac{1}{\beta^2}\right)}(\lambda_8^2 + \psi_8^2)\frac{\text{ohms}}{\text{wavelength}} \quad (17b)$$

where $(\lambda_7 + j\psi_7)$ is the complex field constant associated with E_z in the dielectric medium and $(\lambda_8 + j\psi_8)$ is the complex field constant associated with H_z.

Fig. 4 shows a comparison of the One-, Two-, and Three-Region results. Fig. 5 shows the variation of R_λ with several of the system parameters.[13] Geometric resonances are evident in Fig. (5c).

[13] A more complete compilation of curves is presented in Rosenbaum.[6]

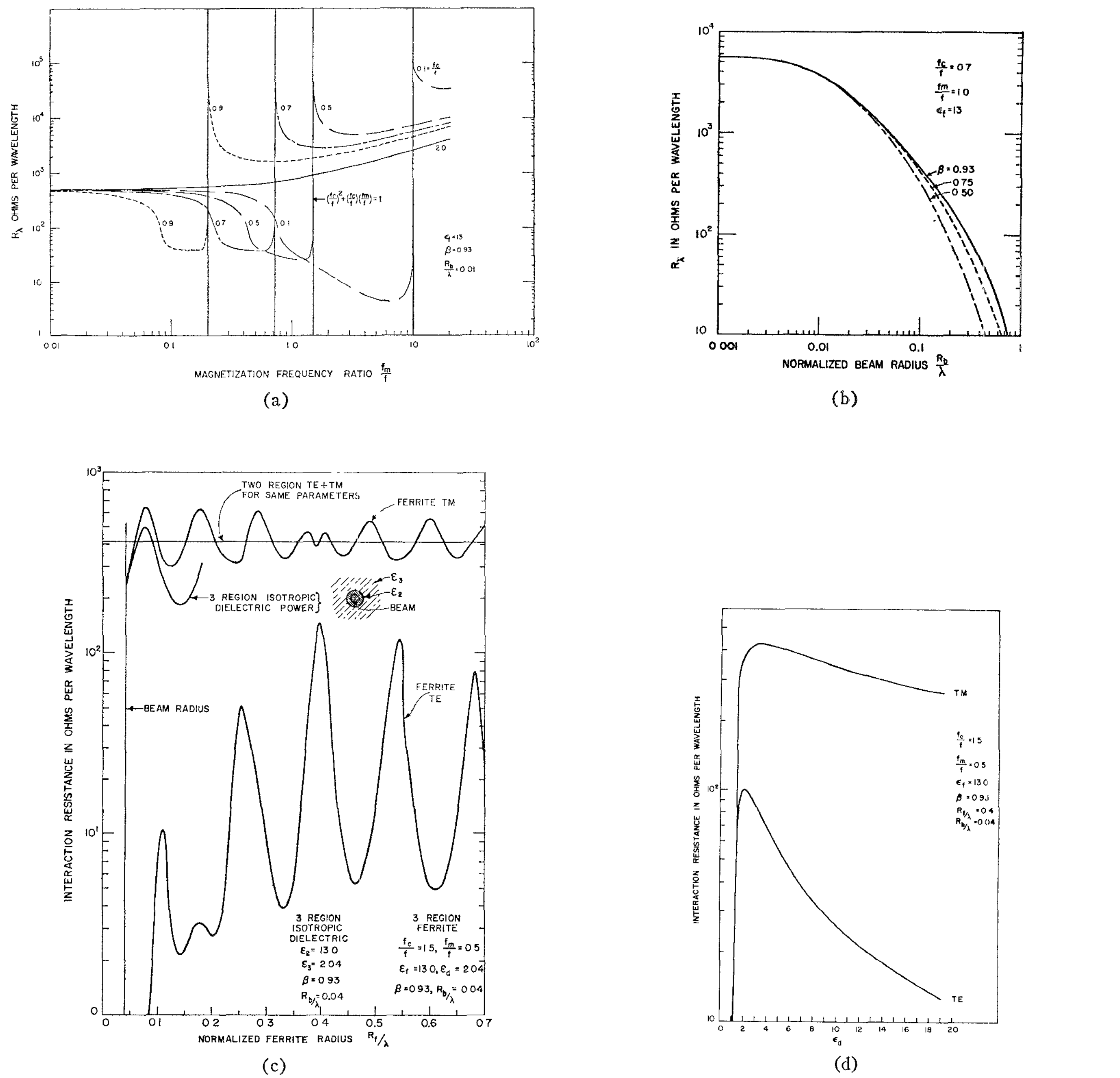

Fig. 5—(a) Two-region lossless interaction resistance as a function of f_m/f. Each vertical line represents the value of f_m/f which satisfies the pole condition $f_c/f(f_c/f+f_m/f)=1$ for the particular value of f_c/f. (b) Two-region lossless interaction resistance as a function of R_b/λ. (c) Geometrical resonances of TE and TM interaction resistances with variation of ferrite radius. (d) TE and TM interaction resistance behavior as outer dielectric constant ϵ_d is varied.

One feature common to these results is the unbounded nature of the interaction resistance when $(f_c/f)^2 + (f_c/f)(f_m/f) = 1$, *i.e.*, when $\mu = 0$. At this point (σ_1) becomes infinite and hence an unbounded longitudinal electric field (E_z) interacts with the beam.[14] Away from this point interaction resistances of the order of kilohms per wavelength are predicted by the lossless theory, as well as the existence of a TE and a TM mode in the outer dielectric.

V. Cerenkov Radiation from Lossy Ferrite

While the assumption of a lossless medium yields results which indicate the general behavior of the Cerenkov radiation generated in it, a more satisfying solution must consider a realistic material, *i.e.*, one which is lossy.

For the sake of simplicity we restrict ourselves to the unbounded ferrite case with a filamentary bunched beam streaming through it. Loss is introduced by setting $\alpha \neq 0$ in the tensor permeability elements. The solution proceeds exactly as in Section IV-A, except that now the radial propagation constants are complex. In order to compute the power in the far field, $(|\sigma_{1,2}r| \gg 1)$, we must assume that losses are small so that those fields remain detectable.

The addition of loss produces coupling between the two modes of propagation. In the lossless case these coupling terms are related as negative complex conjugates and so their real parts do not contribute to the interaction resistance. In the lossy case their contribution is small and may be neglected. The effect of loss[16] (see Fig. 6) is to limit the power generated to a finite value and to shift the maximum interaction to higher magnetic fields.

One may deduce the three-region lossy behavior from this treatment. The power radiated must be bounded; the peak interaction will shift to higher magnetic fields, and any geometric resonance will tend to be damped.

VI. Experiment

The problem of simulating an unbounded dielectric was successfully solved by Enderby[3,17] with his isotropic Cerenkov coupler/radiator in which TM power extracted from the beam propagates at the Cerenkov

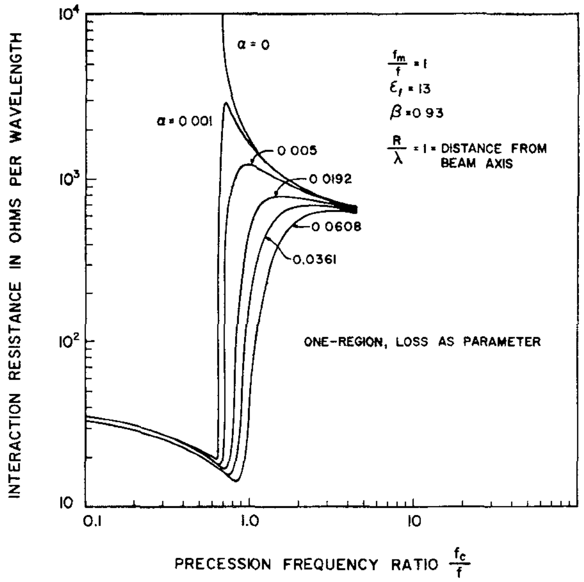

Fig. 6—Lossy one-region interaction resistance as function of f_c/f.

angle $(\cos \theta_c = 1/\beta\sqrt{\epsilon_d})$ to an air-dielectric interface where it is totally internally reflected. This reflected wave then reaches a second interface at the Brewster angle where it is totally transmitted into free space, parallel to the beam direction. This is shown schematically in Fig. 7.

In this experiment a ferrite tube is placed concentric with Enderby's cone structure and immersed in a dc magnetic field. A bunched electron beam, produced by the Rebatron[18] (Relativistic Electron Bunching Accelerator), fills the hole in the ferrite tube. The bunches travel at velocity 0.93 c_0. The peak dc current is about 20 ma and the entire system is pulsed at 60 cps.

As described previously, a TM_{01} and a TE_{01} mode exist in this dielectric cone. Since no Brewster angle exists for the TE_{01} mode only part of its power is transmitted. If we neglect dielectric losses in the Teflon cone, 100 per cent of the TM_{01} power and 88 per cent of TE_{01} power entering it are radiated in a beam parallel to the cone axis. Negligible amounts of reflected power return to the beam axis. The TM_{01} power has at this point been converted into a TEM mode which radiates into the coaxial horn.

The microwave energy is radiated into a coaxial horn which may be connected to either a coaxial line cut off for TE_{01} propagation, or a coaxial line with TEM mode suppressors. It is thus possible to measure either TE_{01} or TEM power in an external waveguide circuit. The coaxial center conductor is used to measure the dc current collected. A schematic view of the entire experiment is shown in Fig. 8.

Experimental values for interaction resistance are determined from

[14] This is an indeterminant situation because, according to Hogan,[15] when $\mu = 0$ the ferrite tends to expel any impinging fields since at this point both the attenuation and propagation constants for a plane-wave in the ferrite are zero (hence zero skin depth). This implies that in the plane-wave case total reflection occurs and that in the Cerenkov case no fields exist in the ferrite for the σ_1 mode waves. The existence of loss resolves this in the real material, as is indicated in Section V.

[15] C. Hogan, "The elements of nonreciprocal microwave devices," Proc. IRE, vol. 44, pp. 1345–1368; October, 1956.

[16] The apparent indeterminacy no longer exists since E_z remains finite and the ferrite allows a certain amount of propagation since μ no longer goes to zero.

[17] C. Enderby, "A Cerenkov radiator for the production of millimeter and submillimeter waves," Wright Air Development Division, Wright-Patterson AFB, Ohio, Tech. Note 1, WADD TN61-1, Contract AF33(616)-7043; January, 1961.

[18] P. Coleman, "Theory of the Rebatron—a relativistic electron bunching accelerator for use in megavolt electronics," *J. Appl. Phys.* vol. 28, pp. 927–935; September, 1957.

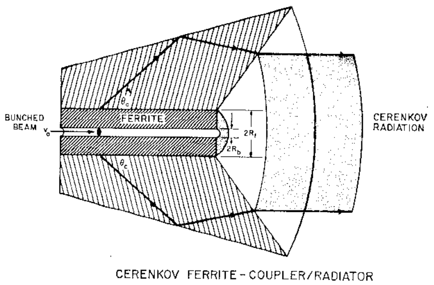

Fig. 7—Cross section of dielectric radiator with ferrite tube inserted.

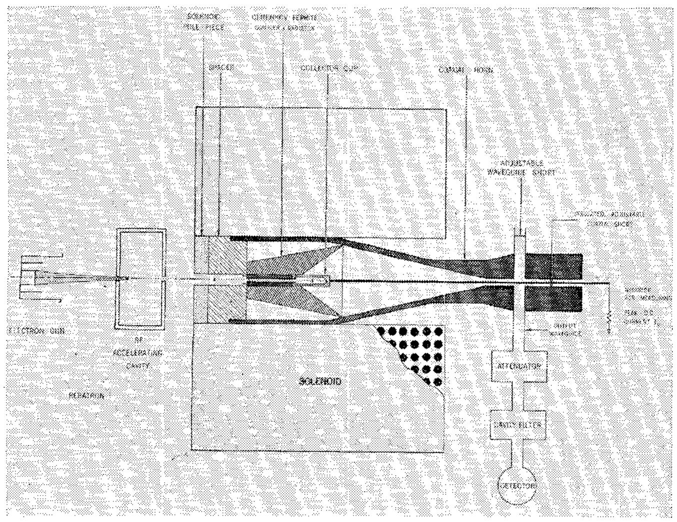

Fig. 8—Schematic view of experiment.

$$R_\lambda = \frac{2P}{(a_n I_0)^2}\left(\frac{\lambda}{L}\right) \quad \text{ohms per wavelength} \qquad (18)$$

where P is the power detected, a_n is the harmonic bunching factor which lies between zero and two, I_0 is the peak dc current, and (L/λ) is number of wavelengths traversed in the interaction.

The value of a_n may be calculated for the case of no magnetic field.[19] The effect of the magnetic field on the beam harmonic content has not been thoroughly examined, but we may assume that at low harmonics a_n is independent of the magnetic field. This assumption seems valid since a_n can change only if some electrons are moved into or out of the region within which all electrons radiate coherently at a given frequency. At low harmonics this region is large and includes most of the electrons in the bunch. Magnetic focusing of the beam (variations in beam current with changing magnetic field) is taken into consideration by computing an interaction resistance, *i.e.*, ($R_\lambda \sim$watts/amp^2). Beam spread (Coulomb debunching) may cause electrons to strike the ferrite and hence not be counted in the current though having contributed to the Cerenkov interaction. This is especially significant at large magnetic field values. Variations in a_n due to the biasing magnetic field and Coulomb debunching have been neglected in the experimental calculations.

VII. Results

Both the theoretical and experimental results are displayed in universal form which is valid at any frequency for which the normalized parameter ratios apply, *e.g.*, $f_m/f=1$. Fig. 9 shows the theoretical TE_{01} and TM_{01} components as functions of f_c/f.

Experimental curves of TE_{01} and TM_{01} interaction resistances are shown in Figs. 10 and 11 along with their respective theoretical lossless curves from Fig. 9. The maximum interaction is, of course, bounded and shifted to higher magnetic field values.

The internal magnetic bias field in the ferrite depends on the applied field and the dimensions and shape of the ferrite sample. If the ferrite is infinite in extent the internal field and the applied field are the same. If, however, the ferrite sample has finite dimensions the internal field is given by the difference between the applied magnetic field and the product of the appropriate demagnetizing factor and the saturation magnetization of the ferrite.

No demagnetization is considered in the theoretical computations since the ferrite is assumed infinite in the longitudinal dimension. In Fig. 9 the assumed value of $f_m/f=1.14$ leads to a value of $f_c/f=0.58$ to satisfy the pole condition $f_c/f(f_c/f+f_m/f)=1$.

This condition is modified in the case of demagnetization such that

$$[f_c/f-(N_z/4\pi)(f_m/f)][f_c/f-(N_z/4\pi)(f_m/f)+f_m/f]=1$$

where $N_z/4\pi$ is the longitudinal demagnetizing factor. Estimating $N_z/4\pi=0.11$ for the sample used in the experiment leads to $f_c/f=0.707$. The value of the minimum observed in Fig. 11 is $f_c/f=0.68$.

In an infinite ferrite medium a strong microwave absorption occurs in the vicinity of precessional resonance ($f_c/f=1$), *i.e.*, the Kittel resonance. In the finite ferrite sample this resonance will be shifted by the demagnetization; so $[f_c/f-(N_z/4\pi)(f_m/f)]=1$ or $f_c/f=0.87$ for the assumed values of f_m/f and $N_z/4\pi$. No such absorption was evidenced in the experimental results.

The ferrite used in these experiments was Ferramic G[20] with the following properties:

Length $=1$ cm	$4\pi M_s=4.54$ kilogauss[8]
O.D. $=0.5$ cm	$\alpha=0.0192$[8]
I.D. $=0.2$ cm	$a_n=0.73$.

[19] M. Sirkis, "The harmodotron—a beam harmonic, higher order mode device for producing millimeter and submillimeter waves," Atomic Energy Commission, Univ. of Illinois, Elec. Eng. Res. Labs., Tech. Rept. 1, Contract AT(11-1)-392, pp. 120–128; September, 1961.

[20] Trade name, General Ceramic Company, Keasbey, N. J.

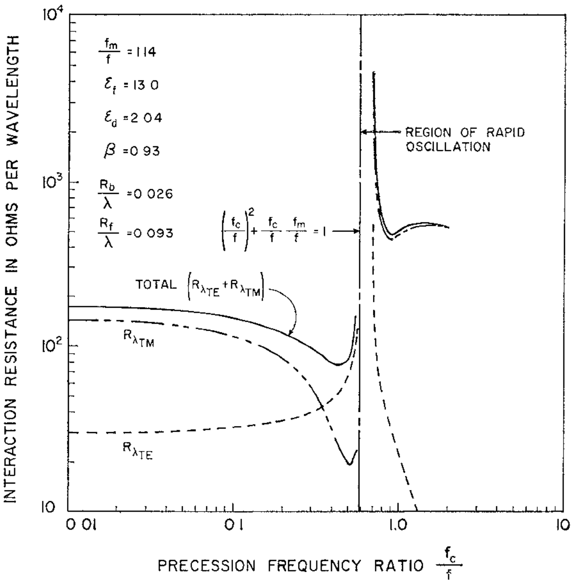

Fig. 9—Three-region TE and TM interaction resistances as functions of f_c/f.

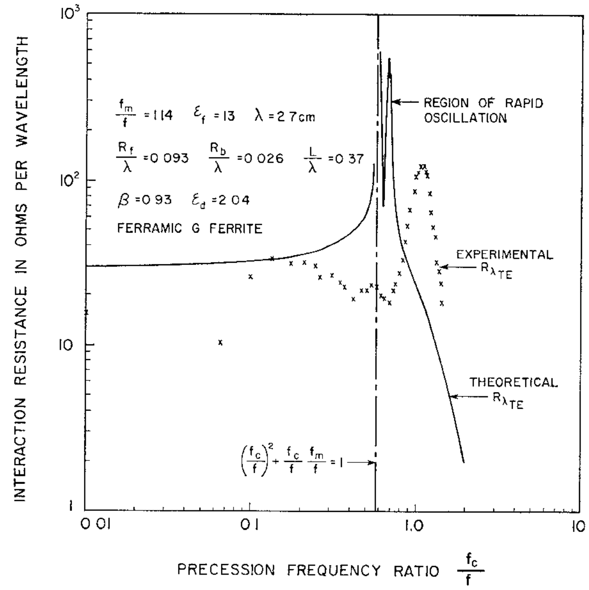

Fig. 10—Comparison of lossless three-region theoretical TE_{01} interaction resistance with experiment.

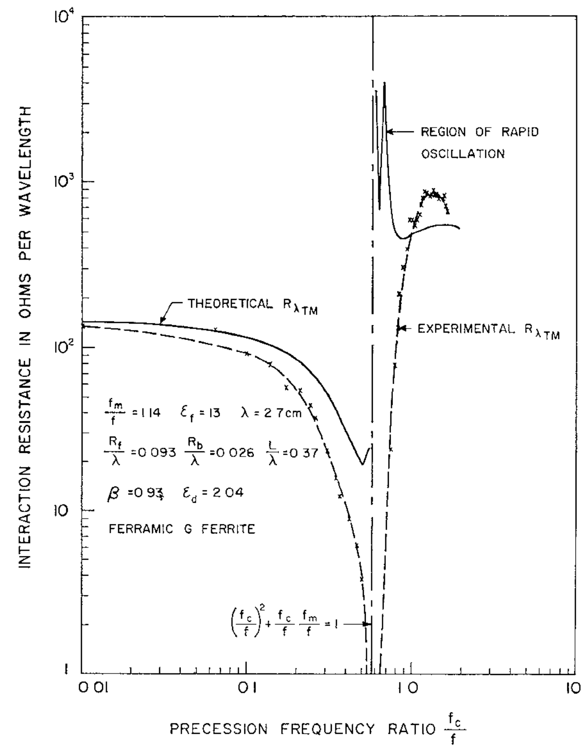

Fig. 11—Comparison of lossless three-region theoretical TM_{01} interaction resistances with experiment.

The frequency used was 11.096 kMc. The experiments were confined to X band in order to use readily obtainable magnetic bias fields.

VIII. Efficiencies

We may define a conversion efficiency as the ratio of the power produced at any given harmonic frequency to the total power available in the beam. The power available in the beam is $P=VI_0=I_0^2R_B$ where V is the potential difference through which the electrons have moved, I_0 is the dc beam current and R_B is the beam resistance. Thus the conversion efficiency is

$$\frac{P_{\text{output}}}{P_{\text{available}}}=\eta=\frac{1}{2}\frac{I_n^2R_\lambda}{I_0^2R_B}\left(\frac{L}{\lambda}\right)=\frac{a_n^2}{2}\left(\frac{L}{\lambda}\right)\frac{R_\lambda}{R_B}. \quad (19)$$

The efficiency of an electron beam-field structure interaction may be thought of in terms of an impedance matching problem. Efficiences will be low so long as the beam resistance is much higher than the interaction resistance as is clearly seen from (19).

The efficiency at any frequency may be increased by

1) Better bunching ($a_n\approx 2$)
2) Extended interaction ($L/\lambda\gg 1$)
3) Increasing the interaction resistance ($R_\lambda\approx R_B$)
4) Lowering the beam resistance ($R_B\approx R_\lambda$).

The Rebatron[18] is an S-band linear accelerator driven by a magnetron providing approximately 300 kw peak power. The rebatron electron bunches achieve a relativistic velocity of 0.93 c_0 which corresponds to an electron energy of 0.88 Mev. We may estimate the beam resistance from $V\approx 10^6$ volts, $I_0\approx 20$ ma, with the result that $R_B\approx 5\times 10^7$ ohms.

An order of magnitude estimate for the efficiency of our Cerenkov device may be obtained by assuming

$$a_n \approx 1$$
$$\left(\frac{L}{\lambda}\right) \approx 0.5$$
$$R \approx 10^3 \text{ ohms/wavelength}$$
$$R_B \approx 5\times 10^7 \text{ ohms.}$$

Then $\eta=0.0005$ per cent.

The estimated power available is 20 kw while the largest measured power was approximately 1 w at 8.33 kMc. This leads to an experimental efficiency an order of magnitude greater than that expected, *i.e.*, $\eta=0.005$ per cent.

Theoretically, an extended interaction ($L/\lambda=10$) and a very low loss ferrite ($\alpha<0.001$) would lead to an efficiency of the order of $\eta=0.50$ per cent. Experimentally as much as 100 w might be expected.

A depressed collector scheme offers the possibility of reducing the beam resistance to perhaps as little as 20 kilohms. Such a device might yield efficiences of the order of 10 per cent. Work along this line has already been reported by Petroff[21] with research in progress by Rowe.[22]

IX. Conclusions

The advantages inherent in the Cerenkov-ferrite device described here are the following:

1) Microwave energy is produced and radiated by the same structure. This structure is suitable for feeding beam type waveguides.

2) The energy is radiated into two independent modes.

3) The structure is broad band in the sense that many beam harmonic frequencies can be extracted using the same device.

4) The level of the interaction is high ($R_\lambda=10^3$ ohms/wavelength) and is easily controllable by changing the dc magnetic bias field.

5) This structure can be made large with respect to radiated wavelengths.

6) It is easy to fabricate.

7) Although the ratio of beam tunnel radius to radiated wavelengths must be kept small for a high interaction, the problem of dielectric charging is largely avoided because of the finite conductivity of the ferrite.

8) The possibility of low velocity electron-beam interactions exists. See Figs. 1 and 5(b).

As well as providing a novel source of microwave energy, the Cerenkov interaction may be useful in analyzing the properties of beams of charged particles, *e.g.*, degree of bunching, as well as the material properties of the interaction medium.

The obvious disadvantage in the application of Cerenkov radiation from ferrites to low millimeter wavelengths is the large values of magnetic fields required to bias the ferrite. For example, to operate in the vicinity of $f_c/f=1$ at $\lambda=1$ mm would require a magnetic field on the order of 10^5 gauss. This fact, coupled with increasing ferrite loss at higher frequencies, will currently limit application to the high millimeter range. However, the hexagonal ferrites and the antiferrimagnetic materials, with their high internal fields, may well find application in extending Cerenkov radiation techniques into the low millimeter range.

Acknowledgment

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[21] M. Petroff, "Efficient Use of Dielectric Cerenkov Couplers for Millimeter Wave Generation," presented at the 20th Conference on Electron Devices Research, at the University of Minnesota, Minneapolis; June, 1962.

[22] J. Rowe, Electron Physics Laboratory, University of Michigan, Ann Arbor, Mich.